# Updatable Strategy Logic

Christophe Chareton Julien Brunel David Chemouil

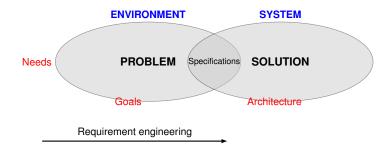
Onera, Toulouse

#### January 8, 2013

### 1 Introduction with language Кн for Requirements Engineering

- 2 State of the art: towards USL
- 3 Syntax and semantics
- 4 Expressive power and model-checking
- 5 Conclusion and future works

# Requirements engineering



- Identify requirements
- Derive specifications from functional requirements

### Кні in three sentences:

- The progressive goals refinement leads to specifications that are expressed in LTL.
- These specifications are gathered into roles (LTL).
- We focus on the problem of a possible assignment of those roles to coalitions of agents.
- Main stakes:
  - Provide sets of specifications that are structured by the agents that have to ensure them.
  - Identify those of these specifications that we cannot ensure with the provided agents.

# Formalism, a first approach with Alternating-Time Temporal logic (ATL: Alur, Henzinger, Kupferman)

### Problem:

- A set  $\mathcal{R}$  of roles and a set  $\Sigma$  of actors.
- An assignment relation  $\subseteq \mathcal{R} \times \Sigma$ .
- Question: for all role  $r \in \mathcal{R}$ , are the concerned agents able to ensure r (LTL)?

### ATL

 ATL enables to express properties of capabilities of agents to ensure temporal properties.

#### $\langle\!\langle {\bf A} \rangle\!\rangle \varphi$

Agents in coalition A are able to ensure the satisfaction of property expressed by φ whatever the other agents do.

## Problems met

Take into account the interaction between coalitions

- Two roles  $r_1$  and  $r_2$ , two coalitions  $A_1$  and  $A_2$ .
- $A_1$  can ensure  $r_1$  but  $A_2$  cannot ensure  $r_2$ .
- Is A<sub>1</sub> able to ensure its role and to enable A<sub>2</sub> to ensure its role at the same time?
- Not expressible in ATL

$$\langle\!\langle A_1 \rangle\!\rangle (r_1 \wedge \langle\!\langle A_2 \rangle\!\rangle r_2)$$

- An agent may be part of several coalitions:
  - If A<sub>1</sub> ∩ A<sub>2</sub> ≠ Ø, then how to express that A<sub>1</sub> and A<sub>2</sub> can ensure their respective roles by playing along a non-contradictory strategy?

$$\langle\!\langle A_1 \rangle\!\rangle r_1 \wedge \langle\!\langle A_2 \rangle\!\rangle r_2$$

Strategy Logic (SL: Mogavero, Murano, Perelli, Vardi )

An observation:

 $\langle\!\langle \mathbf{A} \rangle\!\rangle \varphi$ 

There is a strategy *x* such that if *A* plays along *x* then  $\varphi$  is ensured.

- Starting idea for SL: separate both elements:
  - A quantifier ((x)): ((x))φ is true iff there is a strategy x such that φ is ensured.
  - A strategy binder (A, x):  $(A, x)\varphi$  is true iff if A plays along strategy for x then  $\varphi$  is ensured.
- Sub-formulas are evaluated in contexts that stores the quantifiers and binders.
- At evaluation of temporals, each agent is bound to a strategy.
- Enables to treat the first problem:

 $\langle\!\langle x_1\rangle\!\rangle(A_1,x_1)([\![x_2]\!](\Sigma\backslash A_1,x_2)(r_1\wedge\langle\!\langle x_3\rangle\!\rangle(A_2,x_3)(r_2))$ 

The second one still holds ...

# Semantics of SL: CGS

Concurrent Game Structures:

Some elements from classical Kripke structures:

- A set of states M
- A set of atomic propositions At
- A valuation function, from M to  $\mathcal{P}(At)$

Transitions:

- A set of agents Σ
- A finite set of possible actions for the agents  $A \subsetneq \mathbb{N}$
- In each state, each agent plays a choice and the transitions are determined by the expressed actions : δ is a function from M × A<sup>Σ</sup> to M.

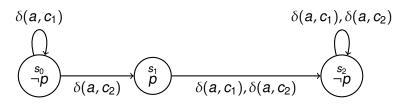




# Semantics of SL: CGS

Concurrent Game Structures:

- Some elements from classical Kripke structures:
  - A set of states M
  - A set of atomic propositions At
  - A valuation function, from M to  $\mathcal{P}(At)$
- Transitions:
  - A set of agents Σ
  - A finite set of possible actions for the agents  $A \subsetneq \mathbb{N}$
  - In each state, each agent plays a choice and the transitions are determined by the expressed actions :  $\delta$  is a function from  $M \times A^{\Sigma}$  to M.



# Semantics of SL: quantifiers and binders

- A strategy is a function  $\sigma$  from  $M^*$  to A
- A *context*  $\kappa$  maps agents and strategy variables to strategies.

### Definition

Satisfaction

•  $\mathcal{M}, \kappa, s \models_{\mathsf{SL}} \langle\!\langle x \rangle\!\rangle \varphi$  iff there is a strategy  $\sigma$  such  $\mathcal{M}, \kappa[x \to \sigma], s \models_{\mathsf{SL}} \varphi$ 

• 
$$\mathcal{M}, \kappa, s \models_{\mathsf{SL}} (a, x) \varphi$$
 iff  $\mathcal{M}, \kappa[a \to \kappa(x)], s \models_{\mathsf{SL}} \varphi$ 

where  $\kappa[a \rightarrow \sigma]$  is obtained from  $\kappa$  by **replacing its value for** *a* **with**  $\sigma$ .

SL uses contexts that do not enable to compose several strategies for an agent

# USL: main ideas

- In SL: when a binder (A, x) occurs, current strategy for A is automatically revoked.
- Aims:
  - either update current strategy without revoking it.
  - either revoke it.
- Means:
  - In general case, a binder (A ▷ x) does not delete the strategies previously bound to A.
  - We make explicit the, perhaps, revocation of strategy: introduction of an unbinder  $(A \not > x)$  expressing it.
  - Delete the constraint for temporals only under complete context.
- Observation: The SL binder (A, x) again is decomposed into two operations
  - Agents in *A* are unbound from their current strategies.
  - They are bound to strategy instanciating *x*.

## Syntax

### Definition

Let  $\Sigma$  be a set of agents, At a set of propositions and X a set of variables, USL( $\Sigma$ , At, X) is given by the following grammar:

State formulas:

$$\varphi ::= p \mid \neg \varphi \mid \varphi \land \varphi \mid \langle \langle x \rangle \rangle \varphi \mid (A \triangleright x) \psi \mid (A \not\models x) \psi$$

Path formulas:

$$\psi ::= \varphi \mid \neg \psi \mid \psi \land \psi \mid \psi \mathsf{U} \psi \mid \circ \psi$$

where  $p \in At, A \subseteq \Sigma, x \in X$ .

Closed formulas are evaluated with no context.

Semantics:

- Structures (NATS)
- Adaptation of the notion of contexts: strategies and plans
- Plan transformations
- Satisfaction relation

### Definition

A Non-deterministic Alternating Transition System (*NATS*) is a tuple  $\mathcal{M} = \langle \Sigma, M, At, \pi, \delta \rangle$  where:

- A set *M* of states, a set At of atomic propositions, a valuation function π, from *M* to P(At), a set Σ of agents.
- A transition function  $\delta : \Sigma \times M \rightarrow \mathcal{P}(\mathcal{P}(M))$ . It maps a pair  $\langle agent, state \rangle$  to a non-empty family of choices of possible next states.
- Choices depend on states and agents.
- $\delta$  directly gives the sets of potential successor.

## Semantics: Strategies and plans

### Definition

- A strategy is a function  $\sigma$  from  $\Sigma \times M^*$  to  $\mathcal{P}(M)$  such that for all  $(a, \tau) \in \Sigma \times M^*, \sigma(a, \tau) \in \delta(a, last(\tau)).$
- A memory μ is a partial function from X to Strat, storing the instantiations for quantified strategies.
- A context κ is a finite list of pairs in (P(Σ) × X), representing the structure of the active bindings.
- A plan  $\Pi$  is a pair of a memory and a context. A plan induces a function from  $M^*$  to  $\mathcal{P}(M)$ :  $(\mu, (A, x))(\tau) = \mu(x)(A, \tau)$  and  $(\mu, \kappa \cdot (A, x))(\tau) =$ 
  - $(\mu,\kappa)(\tau) \cap \mu(x)(A,\tau)$  iff it is not empty,
  - else (μ, κ)(τ)

The semantics also uses the following transformations for a context:

• A plays 
$$x : \kappa[A \to x] = \kappa \cdot (A, x)$$

A revokes x:

$$(A_1, x)[A \rightarrow x] = (A_1 \setminus A, x) (\kappa \cdot (A_1, x))[A \rightarrow x] = \kappa [A \rightarrow x](A_1 \setminus A, x)$$

Quantifier:

for all 
$$x_i$$
 in  $dom(\mu) \setminus \{x\}, \mu[x \to \sigma](x_i) = \mu(x_i)$ 

$$\blacksquare \ \mu[x \to \sigma](x) = \sigma.$$

# Semantics: satisfaction

### Definition

Let M be a *NATS*, then for all memory  $\mu$ , context  $\kappa$  and state s,

- $\mathcal{M}, \mu, \kappa, s \models \langle\!\langle x \rangle\!\rangle \varphi$  iff there is a strategy  $\sigma \in Strat$  such that  $\mathcal{M}, \mu[x \to \sigma], \kappa, s \models \varphi$
- $\mathcal{M}, \mu, \kappa, \mathbf{s} \models (\mathbf{A} \triangleright \mathbf{x})\varphi$  iff for all  $\lambda$  in  $out(\mu, \kappa[\mathbf{A} \rightarrow \mathbf{x}]), \mathcal{M}, \mu, \kappa[\mathbf{A} \rightarrow \mathbf{x}], \lambda \models \varphi$
- $\mathcal{M}, \mu, \kappa, \mathbf{s} \models (\mathbf{A} \not\models \mathbf{x})\varphi$  iff for all  $\lambda$  in  $out(\mu, \kappa[\mathbf{A} \twoheadrightarrow \mathbf{x}]), \mathcal{M}, \mu, \kappa[\mathbf{A} \twoheadrightarrow \mathbf{x}], \lambda \models \varphi$

Let  $\varphi$  be a closed formula, then  $\mathcal{M}, \mathbf{s} \models \varphi$  iff  $\mathcal{M}, \mu_{\emptyset}, \kappa_{\emptyset} \models \varphi$ .

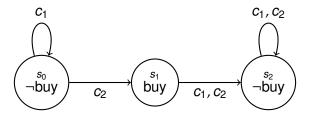
#### The second problem from KHI is resolved:

If  $A_1 \cap A_2 \neq \emptyset$ , can  $A_1$  and  $A_2$  ensure their respective roles by playing along a non-contradictory strategy?

$$\langle\!\langle x_1\rangle\!\rangle (A_1 \triangleright x_1)(r_1 \land \langle\!\langle x_2\rangle\!\rangle (A_2 \triangleright x_2)r_2)$$

# Expressive power: Sustainable capability

- A notion of *sustainable capabilities*:
- A capability for an agent that remains active even if already employed.
- Intuitive example: Alice can always buy car
  - She can buy a car once and decide when
  - In SL:  $\langle x_1 \rangle (a, x_1) \Box (\langle x_2 \rangle (a, x_2) \circ buy)$
  - In USL:  $\langle x_1 \rangle (a \triangleright x_1) \Box (\langle x_2 \rangle (a \not > x_1) (a \triangleright x_2) \circ buy)$

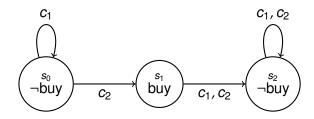


- True at s<sub>0</sub> by strategy allways-c<sub>1</sub>
- She can remain able to buy it, but only provided she never does.
- Her capability to buy a car is not sustainable.

## Expressive power: sustainable capability

Intuitive example: Alice can always buy car

- She can buy as many as she wants whenever she wants:
- In USL:  $\langle\!\langle x_1 \rangle\!\rangle (a \triangleright x_1) \Box (\langle\!\langle x_2 \rangle\!\rangle (a \triangleright x_2) \circ buy)$

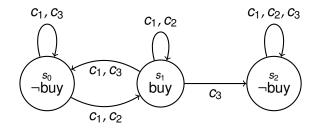


false at  $s_0$  since contraditory strategies.

### Expressive power: sustainable capability

Intuitive example: Alice can always buy car

She can buy as many as she wants whenever she wants:
In USL: ((x<sub>1</sub>))(a ▷ x<sub>1</sub>)□(((x<sub>2</sub>))(a ▷ x<sub>2</sub>) ○ buy)



true at  $s_0$ :

- any occurrence of  $c_2$  from  $s_0$  or  $s_1$  buys a car.
- always-c<sub>1</sub> enables to maintain the capability.
- always-c<sub>1</sub> is not contradictory with any occurrence of c<sub>2</sub>

#### Theorem

There is a transformation of CGS  $\mathcal{G}'$  to NATS  $\mathcal{G}'$  and from formulas  $\theta$  in SL to formulas  $\theta'$  in USL such that for all  $\theta \in$  SL and for all CGS  $\mathcal{G}, \mathcal{G} \models \theta$  iff  $\mathcal{G}' \models \theta'$ . Furthermore, upon SL{1}, this transformation reduces to the actions-choices equivalence.

#### Theorem

There is a formula in USL{1} not expressable in SL{1}.

We proved the second theorem with formula  $\Gamma_{\infty} := \langle\!\langle x \rangle\!\rangle (a \triangleright x) \Box (\langle\!\langle y \rangle\!\rangle (a \triangleright y) \circ p \land \langle\!\langle y \rangle\!\rangle (a \triangleright y) \circ \neg p)$ . It asserts that *a* is sustainably able to decide whether *p* holds or not in next state.

#### Theorem

- The model-checking of USL is NONELEMENTARYTIME decidable.
- The model-checking of USL under memoryless strategies (USL<sup>0</sup>) is **PSPACE**-complete.

### A formalism that:

- Enables composition of strategies for one agent and the sustainable capabilities.
- Unifies it with the classical branching-time mechanisms of strategies' revocation.
- Uses strategies that are both updatable and revocable.
- Holds similar model-checking results as comparable formalisms (SL, ATL<sub>sc</sub>, Brihaye, Da Costa, Laroussinie, Markey)

### Expressive power:

- Express sustainable capabilities as fixed points properties, compare USL with extensions of μ-calculus dealing with strategies (*QD*<sub>μ</sub>, S. Pinchinat).
- Further explore the possibilities enabled by free use of the unbinder.

### Related to KHI:

- Further criteria for model correctness: ensure a role *rl* assigned to an actor *a* does not contradict its pursued goals.
- Compare the efficiency of different strategies in case they do not fully ensure the satisfaction of the roles.

Thank you for your attention

Any question?